

ANIMADVERSIONS

ON

DR. STEWART'S COMPUTATION

OF THE

SUN'S DISTANCE FROM THE EARTH.

A. A. M. A. B. V. R. S. I. O. N. S.

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DR. STEWART'S COMMUTATION



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BY JOHN LANDEN, F. R. S. *K*

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PRINTED FOR THE AUTHOR,

BY GEORGE BIGG, SUCCESSOR TO MR. DRYDEN LEACH;

AND SOLD BY J. NOURSE, IN THE STRAND, BOOKSELLER TO HIS MAJESTY.

MDCC LXXI.

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ANIMADVERSIONS
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DR. STEWART'S COMPUTATION
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DR. *Stewart* has told the world, that he has "ascertained the solar force affecting the gravity of the moon to the earth, and from that has calculated *very accurately* the mean distance of the sun from the earth." Now, having examined what the *Doctor* has done; and having found, not only his principles very exceptionable, but also his calculation egregiously erroneous; I cannot, as the subject is of importance, unconcernedly observe *Error* promulgated as *Truth*, but must, as a friend to science, take up my pen and point out the faults I have discovered.

By what follows it will appear, that a conclusion different from the *Doctor's* may be obtained by following his own method, varying the steps a little, yet taking none but such as will undoubtedly bring us as near the truth as those taken by him.

The sun moon and earth being at S, L, and T respectively, (see *fig. 1.*) and the square of SL being to the square of ST as ST to LE, if ST re-

B

present

present the force of the earth to the sun, LE will represent the force of the moon to the sun; let the force LE be resolved into the forces LF , LG : because TS , FL are equal and parallel, the forces TS , FL , do not disturb the motion of the moon and earth with respect to one another; there remains therefore, in ascertaining the disturbing force of the sun upon the moon, only the force LG to be considered. Draw GN perpendicular to TL , meeting TL in N ; and complete the parallelogram $LNGO$; the force LG may be resolved into the forces LO , LN , and the force LO may be altogether neglected, because, when the moon is at the same distance from the point A , on the other side of TA , its effect is quite opposite and equal, and exactly balances the other effect; so that this force does not affect the periodic time of the moon round the earth; there remains therefore only the force LN to be considered.

In ST take TP on the same side the center T with S ; and let the rectangle STP be equal to the square of TA ; bisect SP in Q , and ST , in R ; and in ST take TK , so, that ST may be triple of TK ; let LM , perpendicular to TA , meet TA in M ; in TA take Tb , so, that the square of TA may be triple the square of Tb ; let LG meet ST in H ; and draw HZ perpendicular to TL , meeting TL in Z .

Because the square of SL is to the square of ST as ST to LE ; and the square of SL is equal to twice the rectangle contained by QM , ST , twice the rectangle contained by QM , ST , will be to the square of ST as ST to LE ; that is, twice QM will be to ST as ST to LE ; therefore QM will be to RT as ST to LE : but SH is to SL as ST , or EG , to EL ; therefore QM will be to RT as SH to SL . Again, because the rectangle STP is equal to the square of TA , ST will be to TA as TA to TP : but because ST , the distance of the sun from the earth, is great in comparison of TA , the distance of the moon from the earth, TA will be great in comparison of TP ; therefore TS , SP , may be considered as equal; therefore QT , TR , may be considered as equal; and therefore QM , MR , may be considered as equal; or, Mm being made equal to PT , QM , mR may be considered as equal; and therefore mR will be to RT as SH to SL : but because TS is great in comparison of TA , or TL , the angle TSL will be little; therefore LS , SM , or S_m , may be considered as equal; therefore

therefore mR will be to RT as SH to Sm ; therefore, inverfely, TR will be to mR as Sm to SH ; therefore Tm will be to mH as RT to Sm : but Sm is nearly double of RT , therefore mH will be nearly double of Tm ; and therefore TH may be confidered as triple of Tm : becaufe ST is triple of TK , and TH triple of Tm , SH will be triple of mK , or triple of MK nearly; therefore MK will be to TK as SH to ST , or EG ; that is, as HL to LG ; that is, as ZL to LN : but the rectangle TLZ is to the rectangle TLN as ZL to LN ; therefore MK is to KT as the rectangle TLZ to the rectangle TLN : and becaufe the angles TZH , TML , are equal, becaufe both right, the rectangle LTZ will be equal to the rectangle HTM ; that is, equal to thrice the rectangle MTm : and becaufe the fquare of TL , or TA , is triple the fquare of Tb , therefore the rectangle TLZ will be triple the excefs of the rectangle MTm above the fquare of Tb ; and therefore MK will be to KT as thrice the excefs of the rectangle MTm above the fquare of Tb to the rectangle TLN ; therefore MK will be to ST as thrice the excefs of the rectangle MTm above the fquare of Tb to thrice the rectangle TLN ; that is, as the excefs of the rectangle MTm above the fquare of Tb to the rectangle TLN : but the rectangle contained by MK , TL , is to the rectangle STL as MK to ST ; therefore the rectangle contained by MK , TL , will be to the rectangle STL as the excefs of the rectangle MTm above the fquare of Tb to the rectangle TLN ; and therefore, alternating, the rectangle contained by MK , TL , will be to the excefs of the rectangle MTm above the fquare of Tb as the rectangle STL to the rectangle TLN ; that is, as ST to LN : and becaufe the centripetal force of the earth to the fun is to the difturbing force of the fun upon the moon at the point L as ST to LN ; therefore the centripetal force of the earth to the fun will be to the difturbing force of the fun upon the moon at the point L as the rectangle contained by MK , TL , to the excefs of the rectangle MTm above the fquare of Tb .

In finding the ratio of ST to LN , we, as well as Dr. *Stewart*, to avoid the inconvenience of retaining furd quantities, have rejected certain quantities on account of their fmallnefs, and have confidered certain other quantities as equal, which are only *nearly* fo. Now, as great caution is requifite in rejecting any quantities in computations, let us compute the
ratio

ratio of ST to LN *exactly*, as we may algebraically; and, by comparing the exact ratio with that which we have found above, and with that which the *Doctor* has found, which are both considered as *nearly* true, let us see whether we have not deviated farther from the truth than he has.

If we write D, d, and x, for TS, TL, TM respectively, LM will be $=\sqrt{d^2-x^2}$, SM=D-x, and $SL=\sqrt{D^2+d^2-2Dx}^{\frac{1}{2}}$; and, F being put for the force of the earth to the sun, $\frac{1}{D^2}$ will be to $\frac{1}{D^2+d^2-2Dx}$ as F

to $\frac{D^2 F}{D^2+d^2-2Dx}$, the force of the moon to the sun; which, when the

value of F is denoted by D, will be $=\frac{D^3}{D^2+d^2-2Dx}=EL$. Now EL

being to EG, or ST, as SL to SH, $\frac{\sqrt{D^2+d^2-2Dx}^{\frac{1}{2}}}{D^2}$ will be = SH;

therefore TH will be $=D-\frac{\sqrt{D^2+d^2-2Dx}^{\frac{1}{2}}}{D^2}$.

Moreover, TL being to TM as TH to TZ, TZ will be $=\frac{Dx}{d}-\frac{\sqrt{D^2+d^2-2Dx}^{\frac{1}{2}}}{D^2} \times \frac{x}{d}$, and $LZ=\frac{Dx}{d}-d-\frac{\sqrt{D^2+d^2-2Dx}^{\frac{1}{2}}}{D^2} \times \frac{x}{d}$.

But LZ is to LN as LH to LG; that is, as LS to LE; that is, as $\sqrt{D^2+d^2-2Dx}^{\frac{1}{2}}$ to $\frac{D^3}{D^2+d^2-2Dx}$; therefore LN will be $=\frac{D^3}{d} \times$

$\frac{Dx-d^2}{\sqrt{D^2+d^2-2Dx}^{\frac{3}{2}}}-\frac{Dx}{d}$: Thus the ratio of ST to LN is exactly ascertained.

By making the comparisons we proposed by means of this last conclusion, it appears, that, in computing the ratio of ST to LN, *Professor Stewart* has deviated farther from the truth than we have in our first computation*. If then we proceed strictly according to his method, without

* It is observable, that our conclusion would have been nearer the truth than the *Doctor's* is, if, in pursuing his method, we had supposed M_m equal to one half, one third, or one fourth, &c. of PT. And it is obvious that a very considerable error arises (pa. 304, Tr. 4.) by inferring from the proportion TR : RM :: MS : SH, that TM will

rejecting any more quantities, our conclusion must be nearer the truth than his is, upon the supposition that his principles are true.

Pursuing as near as possible the steps Mr. *Professor Stewart* has taken in his 8th, 9th, and 10th, *Prop. Traß 4*, we find that the *mean* disturbing force of the sun upon the moon will be determined as follows. Let the circle ABCD whose center is T represent the orbit of the moon round the earth at T; let AC be the syzigial line, and BD the line of the quadratures; in TA take Tb, Te, so, that the square of TA may be triple the square of Tb, and the square of Tb double the square of Te; produce TA to K; and let TK represent one third part of the distance of the sun from the earth; produce TC to P; and let TP be equal to TK; in TK take TE, such, that the square of TE may be to the square of TA in a ratio compounded of the ratio of the rectangle KeP to the rectangle KAP and of the square of TK to the rectangle KbP; and in TB take TF, such, that the square of TB may be to the square of TF, as the rectangle KbP to the rectangle KeP; with the semiaxes TE, TF, let there be an ellipsis EFGH described; take TQ, such, that the rectangle KbP may be to the rectangle KTC as the square of TA to the square of TQ: the force of the earth to the sun will be to the *mean* disturbing force of the sun upon the moon as the circle TQ to the ellipsis EFGH — the circle ABCD; that is, as the square of TQ to the excess of the rectangle ETF above the square of TA.

Fig. 2.

Though what we have already done, with what is done by Dr. *Stewart* in the 9th *Prop. of his Suppl. to his Traßs*, be sufficient for the obtaining an algebraic equation, from whence D may be determined, yet let us follow

will be to MH as RT to SM, and that therefore (SM being nearly double of RT) HM will be nearly double of TM: for the first proportion being not exactly true, but only nearly so on account of the greatness of the quantities retained in comparison of the quantities neglected; the differences TR — RM and MS — SH (which are equal to TM, MH, respectively) will not in general be nearly in proportion to each other as RT to SM; because though the neglected quantities be small in comparison of RT, RM, MS, or SH, they are not small in comparison of HM or TM. HM is so far from being in general double of TM, (as Dr. *Stewart* makes it,) that TM is sometimes greater than HM; even a hundred, a thousand, or ten thousand, &c. times greater!

C

that

that Gentleman a little farther, and take notice of the construction which our conclusion suggests to us, (which is nearly similar to his in Prop. 22. Tr. 4,) and the rather as it will facilitate the numerical calculation of the value of D, the required distance of the sun from the earth.

Fig. 3. Let the circle whose diameter is AB and center T represent the orbit of the moon round the earth at T; in TA produced, let TV be taken, so, that TV may be to TA in the duplicate ratio of the periodic time of the earth round the sun to the periodic time of the moon round the earth; take CT, so, that TV may be to TC as the force of the moon to the earth to the *mean* disturbing force of the sun upon the moon; and take $\overset{\#}{TC}$ equal to AC; let the rectangle $\overset{\#}{CTD}$ be equal to twice the square of AT; let $\overset{\#}{AG}$ be equal to BG, and such, that twice the rectangle contained by AG, $\overset{\#}{DG}$, may be equal to thrice the rectangle ATD; draw GH perpendicular to AB meeting the circle in H; join TH, and draw TE perpendicular thereto meeting the circle in E; draw EK a tangent to the circle meeting TA produced in K; the mean distance of the moon from the earth being represented by AT, KT will represent one third part of the distance of the sun from the earth; and therefore GH will be to thrice TH, or thrice TA, as the mean distance of the moon from the earth to the mean distance of the sun from the earth.

NUMERICAL CALCULATION.

$$TA : TV :: 1 : 178.725;$$

$$TV : TC :: 1 : 0.002797722, \text{ by the } Do\text{ctor's } 27th \text{ Prop. Tr. 4.}$$

$$TA : TC :: 1 : 0.50002286445;$$

$$TA : \overset{\#}{TC} :: TA : TA - TC :: 1 : 0.49997713555.$$

$$TD : TA :: 2TA : \overset{\#}{TC} :: 2 : 0.49997713555.$$

Considering TD as = 2, TA will be = 0.49997713555,

$$DA = 1.50002286445, DB = 2.49997713555,$$

$$\text{the rectangle } ATD = 0.9999542711 = \frac{2}{3}AG \times \overset{\#}{DG} = \frac{2}{3}AG \times$$

$$\overline{DB - AG}.$$

Whence

$$\text{Whence } AG = \frac{DB}{2} - \frac{\overline{DB}^2}{4} - \frac{3}{2} ATD \Big|^{\frac{1}{2}} = 0.999908554743.$$

Now 2 TA being = 0.9999542711,

BG will be = 0.000045716357,

AG × BG = 0.000045712176455985031251,

and $\overline{AG \times BG}^{\frac{1}{2}} = 0.006761078054 = GH.$

Consequently GH : 3 TA :: 1 : 221.8*.

Thus the distance of the sun from the earth appears to be not *half* so great as *Doctor Stewart* makes it!

I shall by the way here just take notice, that all Mr. HORSLEY has written on this subject in the *Philos. Transact.* for the year 1767 amounts to no more than computing numerically from Dr. *Stewart's* construction, as I have now done from the construction given above; therefore, the *Doctor's* error arising from the steps he had taken before he inferred his construction, Mr. HORSLEY's conclusion is as wide of the truth as the *Doctor's* is.

The *Doctor*, in his 27th *Prop. Tr.* 4, reckons that (*q*) the time in which the moon wou'd describe a circle round the earth at the distance TA by the action of the earth alone, is to (*p*) the time in which the moon revolves round the earth by the joint forces of the earth and sun in the subduplicate ratio of *Tm* to TA; that is, as $\overline{f - nf}^{\frac{1}{2}}$ to $f^{\frac{1}{2}}$, AT being expressed by *f*, and *Am* (= TA - *Tm*) by *nf*. Whence we learn, that the *Doctor* conceives *q* and *p* to be respectively equal to the periodic times of two bodies describing equal circles by the action of the centripetal forces *f* and *f - nf*; the periodic times in equal circles being reciprocally as the square roots of the centripetal forces respectively. Therefore, if, in *Prop.* 14, *Tr.* 4, *p* be the periodic time in the circle ABCD, *f - nf* shou'd

* This value of D is greater than the value thereof resulting from the same principles when the calculation is more accurate.

If we had supposed *Mm* equal to one half, or one third &c. of PT, the value of D wou'd have come out greater than this, but less than what Dr. *Stewart* makes it.

according

according to the above observation, be there considered as the force of the moon to the earth; and therefore, in the consequent calculation, nf being the *mean* disturbing force of the sun upon the moon, $f - nf$ shou'd be consider'd as the force of the moon to the earth. But the Doctor in his 27th *Prop. Tr.* 4, and in his *Suppl.* has considered AT , or f , as the force of the moon to the earth; Am , or nf , being the *mean* disturbing force of the sun upon the moon: and having computed that AT is to Am as 357.43365 to 1, he afterwards makes use of that proportion as the ratio of the force of the moon to the earth to the *mean* disturbing force of the sun upon the moon, instead of the proportion 356.43365 to 1, or $f - nf$ to nf . The error thus occasioned is vastly great; for correcting the *Doctor's* calculation by using the proportion 356.43365 to 1, instead of 357.43365 to 1, as the ratio of the force of the moon to the earth to the *mean* disturbing force of the sun upon the moon, the distance of the sun from the earth, comes out *no more than* $62.8 \times d!$ and by so correcting our calculation D comes out equal to *but* $28.09 \times d!$

If we calculate still more accurately, as we readily may by a method much more suitable for the purpose than that which the *Doctor* has made use of, that Gentleman's inaccuracy will appear still greater.

We have found that LN (Fig. 1.) is exactly equal to $\frac{D^3}{d} \times \frac{Dx - d^2}{D^2 + d^2 - 2Dx}^{\frac{3}{2}} - \frac{Dx}{d}$, which is $= \frac{3x^2 - d^2}{d} + \frac{15x^3 - 9d^2x}{2Dd} + \frac{3d^4 - 30d^2x^2 + 35x^4}{2D^2d} \&c.$ Half the sum of the disturbing forces at opposite points L, l , is therefore to the force of the sun upon the earth as $\frac{3x^2 - d^2}{Dd} + \frac{3d^4 - 30d^2x^2 + 35x^4}{2D^2d} \&c.$ to 1.

And it is plain, that, agreeable to the *Doctor's* idea of a *mean* disturbing force, the force of the sun upon the earth will be to the *mean* disturbing force of the sun upon the moon as the fluent of $\frac{d \dot{x}}{\sqrt{d^2 - x^2}}$ generated whilst x from

x from ϕ becomes equal to d , to the fluent of

$$\frac{dx}{\sqrt{d^2 - x^2}} \times \frac{3x^2 - d^2}{Dd} + \frac{3d^4 - 30d^2x^2 + 35x^4}{2D^3d} \&c. \text{ generated in the}$$

same time. The ratio of which fluents being as 1 to $\frac{d}{2D} + \frac{9d^3}{16D^3} \&c.$

$F \times \frac{d}{2D} + \frac{9d^3}{16D^3} \&c.$ will be $= f \times 0.002797722$, f denoting the centripetal force of the earth upon the moon; for by *Professor Stewart's* 27th *Prop. Tr. 4*, the mean disturbing force of the sun upon the moon is $= f \times 0.002797722$, which we denote by nf .

But P and p being respectively put for the periodic time of the earth round the sun and the periodic time of the moon round the earth, P^2 (according to the observation made in pa. 7) will be to p^2 as $\frac{f - nf}{d}$ to $\frac{F}{D}$; therefore $\frac{P^2 F}{D}$ will be $= \frac{p^2 \times f - nf}{d}$; and therefore it is evident

$$\frac{1}{2} + \frac{9d^2}{16D^2} \&c. \text{ will be } = \frac{nP^2}{1 - n \times p^2}.$$

Hence D is found $= 19.86 \times d$, which is not one twenty fourth part of the value of D as computed by *Professor Stewart* from the same principles!

If we had considered P^2 to p^2 as $\frac{f}{d}$ to $\frac{F}{D}$, D would have come out $= 156.8 \times d$,

But there is yet another very considerable fault in the *Doctor's* calculation, occasioned by his not being cautious enough in rejecting quantities in his 27th *Prop. Tr. 4*, where, to render his conclusion more simple, he erroneously considers TG , Ta , Tm , and TP , Tx , TA , as two series of proportionals; which are not such; and, in consequence of his so doing, the value of the mean disturbing force of the sun upon the moon is by him, in the 9th *Prop. of the Suppl. to his Tracts*, brought out too little. Instead of $f \times 0.0027977$, which is the result of his calculation, it should be $f \times 0.0028016$; as appears by his own reasoning, if we avoid the error just now mentioned.—For, by *Tr. 4*, pa. 379, the

D

time

time (t) the moon takes to revolve from apogee to apogee, is to the time (q) in which the moon would describe a circle round the earth at the distance TA (Tr. 4, Fig. 111.) by the action of the earth alone, as the rectangle contained by Ay, Tx, to the square of TA; that is, $t : q ::$

$$f^2 \times \frac{1-4n}{1-5n} : f^2; \text{ TA being } =f, \text{ AE} =nf, \text{ and Ay} \times \text{Tx} =$$

$$\frac{f}{1-5n} \times \frac{f \times 1-4n}{1-5n}. \text{ But, by what is done in the same Tract}$$

pa. 381, $q : p :: \sqrt{f-nf} : f^{\frac{1}{2}}$. It follows therefore, that $t : p ::$

$$\frac{1-4n}{1-5n} \times \frac{1-n}{1-5n} : \frac{1-5n}{1-5n} : \text{ from whence it appears that } n \text{ is } =0.0028016.$$

If the last mentioned expression for the *mean* disturbing force be made use of instead of that which we before obtained by means of the *Doctor's* calculation, we shall find, by the most exact computation, D come out equal to *but* $16.27 \times d!$ which is not *one thirtieth part* of Dr. STEWART'S value of D!

If, *ceteris paribus*, we consider P^2 to p^2 as $\frac{f}{d}$ to $\frac{F}{D}$, D will be found $=28.02 \times d.$

Notwithstanding the sufficiency of the above proportion for determining n , yet the *Doctor* makes use of farther means for that purpose.—The quantities TG, Ta, Tm, and TP, Tx, TA, (whose values are $f-5nf$, $f-3nf$, $f-nf$, and $f \times \frac{1-3n}{1-5n}$, $f \times \frac{1-4n}{1-5n}$, f , respectively,) being considered as two series of proportionals, he has

$$f-5nf : f-3nf :: f-3nf : f-nf,$$

$$\text{and } f \times \frac{1-3n}{1-5n} : f \times \frac{1-4n}{1-5n} :: f \times \frac{1-4n}{1-5n} : f;$$

and these proportions, which are both palpably false, (n in each being $=0$), he compounds with the proportions

$$t : q :: f^2 \times \frac{1-4n}{1-5n} : f^2,$$

and

and $q : p :: \sqrt[n]{1-n} : \sqrt[n]{1-3n}$, taken notice of above : by which means he deduces

$$r^3 : p^3 :: TP^3 : TA^3 :: \frac{1-3n}{1-5n} : 1^3, \text{ where } n \text{ is } = 0.002797722.$$

Thus we see how that Gentleman (more nice with regard to the simplicity of his conclusion than the truth of it) deduces, instead of our proportion where n is $= 0.0028016$, another less exact, by a specious shew of reasoning, without considering how much the value of D would be affected by a small alteration in the value of n !

It appears by what is done above, that Dr. Stewart (admitting his principles to be true) has committed *three errors* in attempting to solve the problem under consideration!

1st. He makes the half sum of the disturbing forces of the sun upon the moon at opposite points L, l , equal to $\frac{D}{d} \times \frac{3x^2 - d^2}{D^2 - 9x^2} \times F$, instead of

$$\frac{D^3x - D^2d^2}{2d \times D^2 + d^2 - 2Dx} - \frac{D^3x + D^2d^2}{2d \times D^2 + d^2 + 2Dx} \times F, \text{ which last expression is the exact value of that half sum.}$$

In consequence of this error, he makes the *mean* disturbing force of

$$\text{the sun upon the moon equal to } \frac{D^2 - 3d^2}{3d\sqrt{D^2 - 9d^2}} - \frac{D}{3d} \times F$$

$$\left(= \frac{d}{2D} + \frac{45d^3}{8D^3} \&c. \times F \right) \text{ instead of } \frac{d}{2D} + \frac{9d^3}{16D^3} \&c. \times F, \text{ which}$$

is the result of the most accurate method of computing it.

* Admitting such reasoning, various other proportions may be obtained for determining the value of n , and consequently various values of D ! for instance, TP being to TA as Ta to TG ; if Tm, Ta, TG be considered as proportionals, TP will be to TA as Tm to Ta ; and therefore

$$r^3 : p^3 :: Tm^3 : Ta^3 :: \frac{1-n}{1-3n} : 1^3,$$

where n is considerably greater than 0.0028016 , and consequently the correspondent value of D considerably less than the value of D as by me above computed.—Admirable reasoning indeed! by which we may prove the sun's distance from the earth to be as great or as little as we please!

It seems to me a little extraordinary, that the *Doctor* should make choice of such an inaccurate method of computation for obtaining such half sum, when (as it appears above) he might have computed the same *quite exactly*. But, I imagine, his choice arose from a singular fondness for his own method, esteeming it more elegant than the algebraic method, without considering, that, however elegant it may be in some instances, it cou'd be of no real use in the computation he proposed to make.

2d. The *Doctor*, I conceive, has erred in considering the force of the moon to the earth, at one time as equal to the centripetal force of the earth upon the moon, and at another time as equal to the excess of the same centripetal force above the *mean* disturbing force of the sun upon the moon.

3d. His conclusion in his 27th *Prop. Tr. 4*, is too inaccurate to answer the purpose for which he intended it, had the rest of his computation been carried on with accuracy.

Correcting the 1st error only, D is found $= 156.8 \times d$;

correcting the 1st and 2d, D is found $= 19.86 \times d$;

correcting the 1st and 3d, D is found $= 28.02 \times d$;

correcting all the three errors, D is found $= 16.27 \times d$.

By Dr. *Stewart's* computation D is found $= 495.93 \times d$!

I do not conclude, that the distance of the sun from the earth is *nearly* equal to the value of D which results from the most accurate calculation above given; being clearly of opinion, that the principles themselves are not to be relied upon.—Of the many objections that may be brought against them, I will mention two or three, which may be sufficient to shew, that an expectation of a conclusion *nearly* true from the *Doctor's* theory wou'd be very far from being well founded.

1st. In the fictitious orbit (Fig. 111. *Tr. 4*.) by means whereof, (I conceive,) the *Doctor* proposed to compute the effect of the *mean* disturbing force of the sun upon the moon in causing the apogee to change its place, the time of moving from apogee to apogee being supposed equal to the *mean* time the moon takes to move from apogee to apogee, the angle described in that time by the apsis in the fictitious orbit will not be

be equal to the *mean* angle described in the same time by the apsis in the lunar orbit : for it appears by computation, that the angle which would be described by a body moving from apogee to apogee in the said fictitious orbit is about $361. 31. 36''^*$; but it appears by observation, that the *mean* angle described by the moon from apogee to apogee is about $363. 4. 7''$. Which inequality, if (contrary to my opinion) it be not altogether an absurdity in the theory, ought certainly to be accounted for; that it may appear, why we are to suppose (as Dr. *Stewart* has done) an equality between the said times rather than between the angles described by the apsides in the two orbits in those times. Apparently the effects of the disturbing forces in the two orbits cannot be properly compared unless these last mentioned angles, which are the measures of those effects, be equal and described in equal times.—Does it not then seem extremely absurd, to admit it as a *principle*, that, although the angular motions be so different, yet the time of moving from apogee to apogee in the fictitious orbit may be considered as equal to the *mean* time the moon really takes to move from apogee to apogee?

2d. The greatest and least distances from the center of force, and some other obvious circumstances in the said fictitious orbit, being not the same as in the lunar orbit, the effect of the *mean* disturbing force, as computed from that fictitious orbit, cannot, I apprehend, be, *any way*, fairly compared with the real effect of the solar force upon the moon; even if we admit that the force LO, Fig. 1, may be neglected. But indeed I might with great reason insist, that the neglecting that force greatly affects the conclusion.

* Vid. NEWT. *Philos. &c.* lib. 1, prop. 45.

Is it not very extraordinary, that the Doctor should omit taking any notice of the measure of this angle?—I can only account for the omission by supposing, that he was not aware of there being any difference between the said angle and the *mean* angle described by the moon from apogee to apogee.

3d. The sun's distance from the earth, as deduced from the *Doctor's* principles by the most accurate reasoning, differing widely from the result of observations, that circumstance alone may perhaps be a sufficient reason for rejecting his theory as false.

To urge in defence of his theory, that the *principles* are true, but that we fail in bringing out our conclusion by reason of our not being able to get the value of the *mean* disturbing force of the sun upon the moon sufficiently exact, is a manifest absurdity; seeing that the more exactly we compute that force (and we may indeed easily compute it much more exactly than the *Doctor*, or his friend Mr. *Horsley*, has done) the more will our conclusion respecting the sun's distance from the earth, deduced from the *Doctor's* principles, differ from the conclusion deduced from the most accurate observations.

I must not omit taking notice, that Mr. *Horsley* (in the *Philos. Transact.* 1769) has declared himself *well satisfied* that there is no error in Dr. *Stewart's* principles.—But as it does not appear, that his declaration is any thing more than his bare opinion unsupported by any argument, I shall only refer him to what is said above.



F I N I S.

E R R A T U M.

Pa. 8, l. 6, *for*, "force of the moon to the earth;" *read*, centripetal force of the earth upon the moon.

Fig. 1.

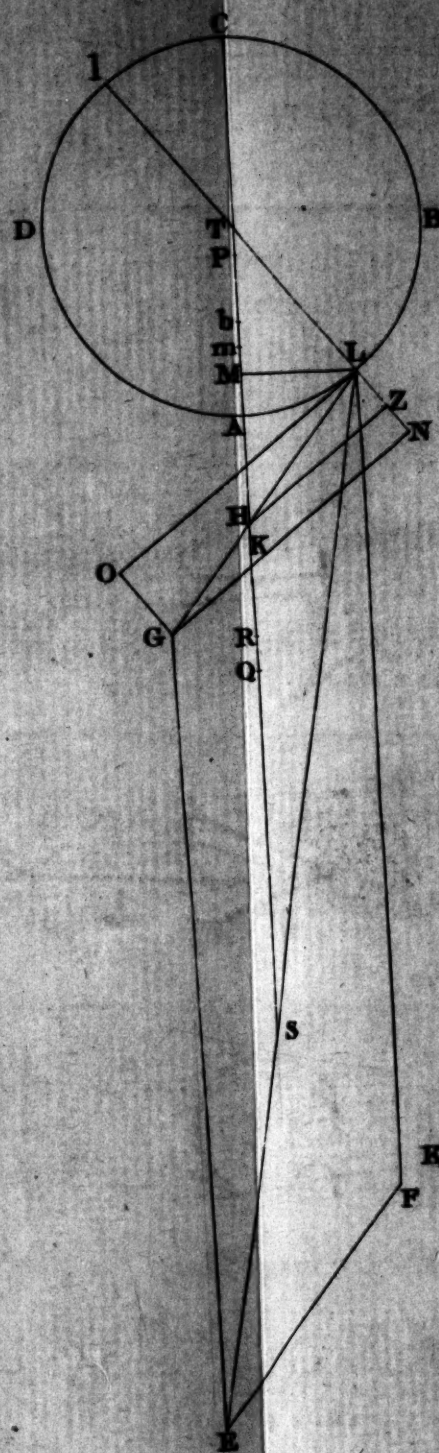


Fig. 2.

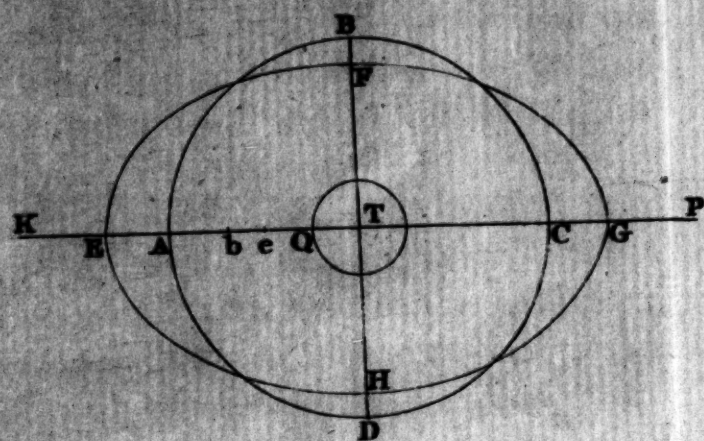


Fig. 3.

